ENGD3008 – Dynamics, M. Goman
Experiment 2 – Simple Gyroscope
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Mechanical Engineering, Year 3
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1. Introduction

The relationship between a spinning disc on a frame and the precession rate of the frame, and the resulting couple, is investigated. The direction of the couple is also deduced by experiment. This is compared with that of theory proposed by I. Newton.

2. Background

2.1 – History

The first description of what is known today as a gyroscope was then described as a phenomenon attributing to toy, a spinning top. But it wasn't until the late 18th century that a gyroscope was used for practical purposes. An English scientist named Searson proposed that a gyroscope could be used to enhance navigational equipment. At the time sailors used sextants for navigation, by measuring the angle between the stars and the horizon. But, in bad weather or poor visibility the horizon was hard to accurately measure. Searson suggested that a gyroscope could be used to attain a level surface to measure from. However, when he set sail to test his theory on the open water the ship sunk and all crew drowned.

2.2 – Basic Principles

A gyroscope can be described as a spinning body, for example a disk or flywheel, which is free to move in any direction about its axis of spin. If the disk is moved from its principle axis it requires a much greater external force (or couple) to displace than it would if it were not spinning. This is attributed to two features, its gyroscopic inertia and its precession.

The inertia of any object is its resistance to movement, and as such the gyroscopic inertia is a consequence of Newton's 1st Law, that a body will continue to move in its direction unless acted upon by an external force. The equivalent of this is a spinning body will continue to spin about its axis unless acted upon by an external couple.

The precession of a gyroscope is the resulting change in direction when a force is acted upon it. Rather than simply following this change in direction it will act at right angles to it. An example is if a body is spinning about an axis on a frame, the result of any torque applied to it is a change in direction at a right angle to the applied torque. Hence if the angular velocity and applied torque are constant the resulting change in direction will cause a conical shape.

2.3 – Newtonian Physics

From the previous description of precession it is clear that the greater the linear velocity (or greater angular velocity) the greater the applied force must be to change its direction of momentum.

Newton described this relationship for a rotating body as

$$\boldsymbol{\omega}_p = \frac{\boldsymbol{Q}}{I\boldsymbol{\omega}_s} \qquad 1$$

Where
$\boldsymbol{\omega}_p$- Angular velocity of the precession, rad/s
$\boldsymbol{\omega}_s$- Angular velocity of the body spinning, rad/s
I – Mass moment of inertia of the rotating body, $kg.m^2$
$\boldsymbol{Q}$ – Applied torque, N.m

3. Apparatus

Gyroscope Unit
2 Speed Control Units

4. Procedure

The gyroscope Unit was checked for structural integrity before any experimentation was conducted. The electrical connections were checked between the speed control units and the variable speed motor for the rotor disc, and the geared motor located underneath the main assembly for the precession.

All external masses were removed from the torque arm to construct the gyroscopes static unloaded position. The torque arm was checked to be balanced by using a marking on the retaining plate, and the moveable balance weight was adjusted so that the torque arm balanced along the vertical axis. This allows accurate base settings.

The safety dome cover was then placed back on the unit.

4.1 - Gyroscopic Couple (Direction)
The power of the speed control units were switched on and the variable speed motor and precession motor were adjusted to spin. The direction of rotation was noted, and the torque arm was noted to either rise or fall.

The precession motor direction was changed, then the variable speed rotor was changed, and then both motors were changed, and at each change in direction the torque arm was observed to either rise or fall.

4.2 - Gyroscopic Couple (Magnitude)
The magnitude of the couple is produced by a combination of the precession and rotor velocity (angular). Therefore to find the magnitude of the couple the rotating speed of the precession is measured for varying rotor disc speeds.

To produce more encompassing results a range of masses are attached to the torque arm, to which end it is theorised that the produced couple will need to be larger when an increased mass is attached.

The procedure is thus

1. The static unloaded position is again checked to ensure balance.
2. A 100g mass is attached to the torque arm.
3. A rotor speed is set using the speed control units.
4. The precession speed is adjusted until the torque arm reaches the same balance in the vertical plane as it did in its static unloaded position.
5. The rotor speed is noted, the precession speed is obtained by timing how long it takes for 10 rotations.
6. Three more results are achieved by repeating steps 3 to 5 for incrementing rotor speeds.
7. The motor is switched off.
8. Steps 2 to 7 are repeated for balance masses of 200g and 300g.

4.3 - Moment of Inertia of Armature
To calculate the couple created by the rotor angular velocity it is necessary to find the inertia of the rotating mass, that is the gyroscope armature. This is found by creating another experiment to measure the armature angular displacement period. This can be assumed as simple harmonic motion and the inertia can thus be deduced.

The procedure to determine the inertia is to hang the armature from a bifilar suspension of wires length L. The assembly is then twisted by about 10°, and when released the time for 50 oscillations is measured. The length L, distance between the wires d, and mass is also measured.

5. Results

5.1 – Gyroscopic Couple (Direction)
By observing the direction of the rotor and precession of the whole gyroscope the couple direction can be seen to act either upwards or downwards. The results show that, if at a specific instant in time the rotor is facing directly toward the observer, then,

If the rotor is rotating clockwise (**CW**) and the precession is spinning **right** (i.e. CW as observed from above it) the couple will act to move the **face** of the rotor **downwards**, as in figure 1.1.

If the rotor is rotating **CW** and precession is going **left** the couple moves the rotor face **upwards**, as in figure 1.2.

If the rotor is rotating anticlockwise (**ACW**) and precession is going **right** the couple moves the rotor face **upwards**, as in figure 1.3.

If the rotor is rotating **ACW** and precession is going **left** the couple moves the rotor face **downwards**, as in figure 1.4.

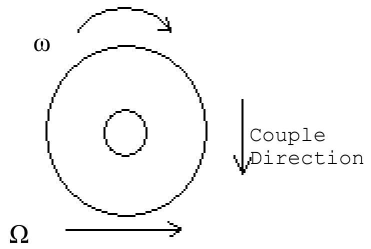

Figure 1.1 – the couple acts downwards if the rotor is facing the observer at an instant in time.

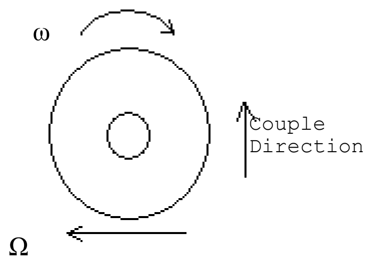

Figure 1.2 – the couple acts upwards if the rotor is facing the observer at an instant in time.

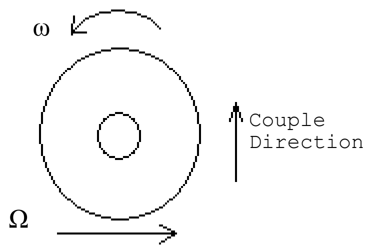

Figure 1.3 – the couple acts upwards if the rotor is facing the observer at any instant in time.

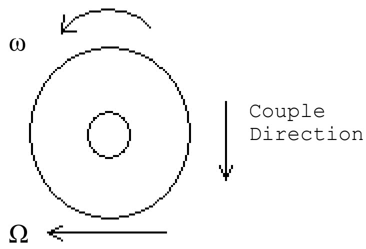

Figure 1.4 – the couple acts downwards if the rotor is facing the observer at any instant in time.

Note $\Omega = \omega_p =$ precession angular velocity

5.2 – Gyroscopic Couple (Magnitude)
Tabulated results from the procedure are shown in Table 1.

Table 1. The results from the experiment

Balance Mass (kg)	Rotor Speed, ω		Precession Speed, Ω	
	(rpm)	(rad/s)	(rpm)	(rad/s)
0.1	495	51.836	48.780	5.108
	833	87.232	28.571	2.992
	1035	108.385	22.642	2.371
	1400	146.608	15.075	1.579
0.2	664	69.534	61.224	6.411
	886	92.782	48.780	5.108
	1056	110.584	41.958	4.394
	1319	138.125	31.746	3.324
0.3	940	98.437	74.074	7.757
	1229	128.701	53.571	5.610
	1464	153.310	45.455	4.760
	1619	169.541	40.000	4.189

These results are used with reference to Eq. 1 to investigate the relationship between the produced couple and the angular velocity of both the rotor and precession.

5.3 – Moment of Inertia of Armature
The inertia described in the background is only applied to the rotating body. In the experiment the rotating body consists of the armature and the rotor assembly. The results from the experiment are shown,

Mass of body, m = 1.09kg
Length of wires, L = 0.53m
Distance between wires, d = 0.073m
Time for 50 oscillations, t = 47.5s

If Newton's 2nd Law of motion considered applicable to this experiment, then

$\boldsymbol{F} = m\boldsymbol{a}$ for a linear motion, the acceleration of a body mass will be proportionate to the applied force. The equivalent for an angular motion is $\boldsymbol{T} = I\ddot{\theta}$, the torque is a product of the inertia of the body and the angular acceleration.

It follows then that the restoring force (or torque) found when rotating the armature is equal to the inertia and angular acceleration. If the angular acceleration is considered representative of simple harmonic motion then the moment of inertia can be determined as

$$I = \frac{m\boldsymbol{g}d^2\boldsymbol{T}^2}{16\pi^2 L}$$

Of units kg.m^2. T is the periodic time, $T = t/50$ = 47.5/50 = 0.95 s.

Therefore,

$$I = \frac{1.09 \times 9.81 \times 0.073^2 \times 0.95^2}{16\pi^2 \times 0.53} = 0.000614, \mathrm{kg.m^2}$$

6. Discussion

6.1 - Gyroscopic Couple (Direction)
Depending on the direction of rotation of the rotor disc and gyroscope unit (i.e. precession direction) the torque arm will either rise or fall due to the direction of the couple. The observation proves to follow the right hand screw rule. If the right hand is open palmed, and the fingers are wrapped around the palm in the direction of the precession rotation, then the thumb (pointing outwards) will show the direction of the acting couple, at a right angle.

6.2 - Gyroscopic Couple (Magnitude)
As previously described the acting couple is the applied torque, $\boldsymbol{\tau}$. The theoretical torque applied can be defined as the relationship,

$$\boldsymbol{T} = \boldsymbol{F}d$$

Where $\boldsymbol{F}$ is the force of the acting couple (i.e. the weight of the applied balance masses) and d is the length of the arm. The torque is found using

$$\boldsymbol{T} = \boldsymbol{\tau} = \boldsymbol{Fd} = \boldsymbol{W}L = m\boldsymbol{g}L \qquad 2$$

Where
m - The balance mass, kg
$\boldsymbol{g}$ – Gravitational constant, 9.81 m/s^2
L – Length of the arm, 0.14m

For example, the acting couple for a mass of 0.1 kg, as in the first experiment is

$\boldsymbol{\tau}$ = 0.1×9.81×0.14 = 0.137 Nm

From Eq. (1) we see that the precession speed is found as a result of the couple, rotor speed and inertia of the body. Rearranging,

$$\boldsymbol{Q} = \boldsymbol{\tau} = I\boldsymbol{\omega}_p\boldsymbol{\omega}_s = I\boldsymbol{\omega\Omega} \qquad 3$$

However, from table 1 it can be seen that the angular velocity of the rotor and precession vary. So, for a balance mass of 0.1kg the applied torque is calculated as,

$$\boldsymbol{\tau}_1 = 6.14 \times 10^{-4} \times 5.108 \times 51.836 = 0.163\ \mathrm{Nm}$$

The results from these calculations are shown in Table 2.

Balance Mass (kg)	Rotor speed, ω (rad/s)	Precession Speed, Ω (rad/s)	Couple, τ (Nm)
0.1	51.836	5.108	0.163
	87.232	2.992	0.160
	108.385	2.371	0.158
	146.608	1.579	0.142
0.2	69.534	6.411	0.274
	92.782	5.108	0.291
	110.584	4.394	0.299
	138.125	3.324	0.282
0.3	98.437	7.757	0.469
	128.701	5.610	0.444
	153.31	4.760	0.448
	169.541	4.189	0.436

Table 2. The acting couple for varying balance masses.

The experiment dictates that the rotor speed is changed, and the precession speed adjusted until the arm is balanced. Hence for a balance mass of 0.1kg the acting couple is averaged for the 4 different rotor and precession speeds the result is τ = 0.156 Nm.

Thus, if Eqs. (2) and (3) are used for all 3 balance masses, and the same procedure is conducted for the average couple, then a plot can be produced comparing the two, as shown in figure 2.

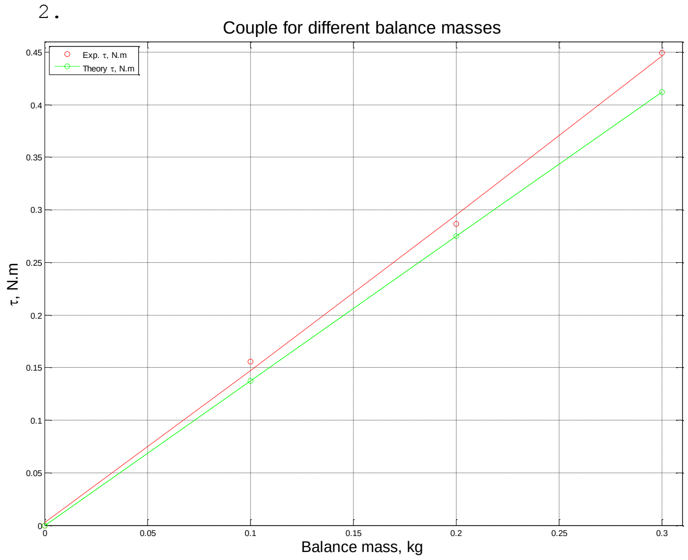

Figure 2. The results from the experimentally and theoretically calculated couple.

From Eq. (3) it is proposed theoretically that the precession and rotor speed have an influence on the couple independently, i.e. if either are increased then the couple will increase proportionately, in a linear fashion, as the equation is only first order. Hence independent plots of the precession and rotor speed are shown,

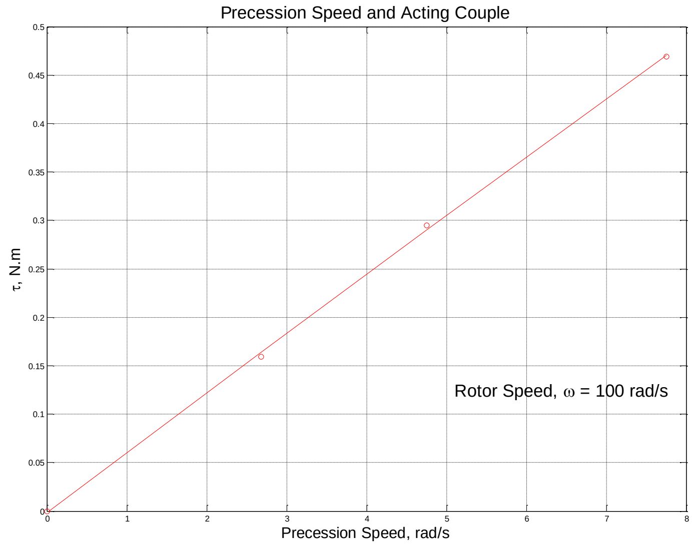

Figure 3. The couple for constant rotor velocity and varying precession velocity.

Note: From table 2 it can be seen that a rotor speed of 100 rad/s for masses 0.1 and 0.2 kg is not recorded, but it is recorded for approx. 90 and 110 rad/s. Hence the precession speed and couple is averaged between these recorded values.

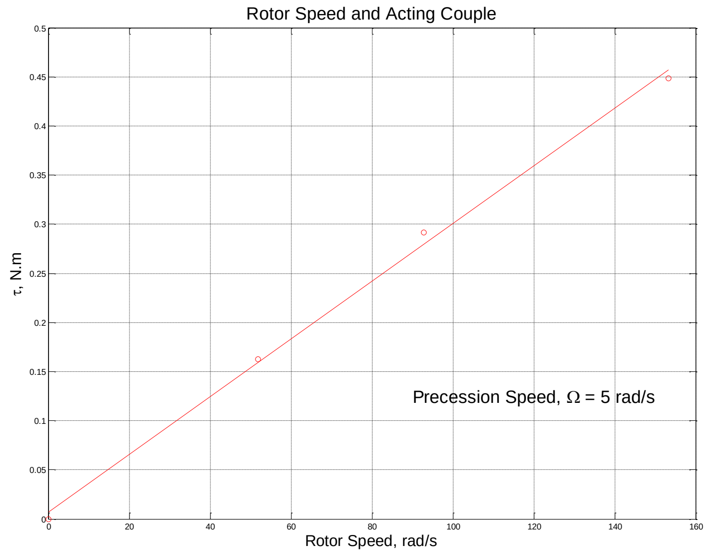

Figure 4. The couple for constant precession and varying rotor velocity.

From figures 3 and 4 it can be seen that the experimental results do show independent influences in a linear manner, but a straight line fit is not exact. This is due to the constant rotor velocity of 100 rad/s and constant precession of 5 rad/s for figures 3 and 4 respectively not being exact. For example the plot of figure 4 uses a precession of 5.1 rad/s, 5.1 rad/s and 4.8 rad/s, as is seen in table 2.

6.3 – Potential Errors

The precession speed is recorded by measuring 10 rotations, but it is inaccurate by the fact that precisely 10 rotations is impossible to measure by observation. It will always be over/under 10 rotations, if even by only minutes or seconds of a degree. Rather, it would be more appropriate to measure 100 rotations, say, or at least a significantly greater amount. Alternatively it could be suggested that a sensor could be attached to a point on the apparatus, with an appropriate signal generator (antenna). For example a magnetic resonance or an eddy current sensor would create a signal (pulse) to detect precisely 1 full rotation, and could be coupled to an accurate timer. This would yield greater accuracy for the precession speed.

It would be inappropriate to suggest here the percentage error caused by manually measuring the precession velocity, although it is shown that for a discrepancy of 1 second the error is 8%.

For example taking the first recording of 12.3 seconds for 10 rotations,

$$\Omega_i = \frac{12.3 \text{ seconds}}{10} \times 60 = 48.780 \text{ rpm}$$

$$\Omega_{ii} = \frac{12.3 + 1 \text{ second}}{10} \times 60 = 45.113 \text{ rpm}$$

$$\text{Percentage Error} = \frac{\Omega_i - \Omega_{ii}}{\Omega_{ii}} \times 100 = 8.13\%$$

6.4 - Applications

The main applications for gyroscopes are used in navigation and guidance systems, and are installed in many types of vehicles, aircrafts, spacecrafts, submarines, guided missiles and ships. Their main advantage in today's technological age is its ability to give vital navigational and velocity information without the need for an external reference, when used with accelerometers and other complementary devices. This is down to its inherent relationship as has been described in Eq. (3), the inertia. Hence due to the system not needing any reference or base parameters to compare with (once initially set) it is more robust and not susceptible to interferences and jamming.

7. Conclusions

- By taking multiple readings of varying rotor speeds and precession for each balance mass an average value for the couple can be determined, and reduces the likelihood of user error.
- As figure 2 shows there is a good relationship between experimental data and that of theory proposed in Eq. (2).
- The right hand screw rule is validated in the experiment, as is shown in figures 1.1 to 1.4.
- The experiment also provided good associations and good first order linear properties with those calculated, as is shown in figures 3 and 4.
- The linear properties contain slight errors due to the angular velocities **not** being precise, i.e. $\omega \approx 100$ rad/s and $\Omega \approx 5$ rad/s for figures 3 and 4 respectively.
- The largest experimental error is due to the limited time when measuring the precession rate.
- Although the principles are quite basic and simplistic, any real world applications are mechanically quite complex (neglecting of course a spinning top).

8. References

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9. Appendix

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%{
Warwick Shipway
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Simple Gyroscope
%}

%Constants
a = (2*pi)/60; % Conversion to rad/s
L = 0.53; % length of the wires, m
d = 0.073; % distance between the wires, m
m = 1.09; % mass of the rotor, kg
t = 47.5; % time for 50 oscillations, s
T = t/50; % time for 1 oscillation, s
g = 9.81; % gravitational accn, m/s^2
arm_length = 0.14; % Length of the torque arm, m

%Moment of Inertia, kg.m^2
I = (m*g*d^2*T^2)/(16*pi^2*L) % kg.m^2

%Exp. Results
%Balance mass
BM = [0 0.1 0.2 0.3]; % kg
%Rotor speed, rpm
Rotor1 = [495 833 1035 1400];
Rotor2 = [664 886 1056 1319];
Rotor3 = [940 1229 1464 1619];
%Precession speed, rpm
Omega1 = (10./[12.3 21 26.5 39.8])*60;
Omega2 = (10./[9.8 12.3 14.3 18.9])*60;
Omega3 = (10./[8.1 11.2 13.2 15])*60;

%Exp. Torque, N.m
Torque1 = I*a^2*(Rotor1.*Omega1)
Torque2 = I*a^2*(Rotor2.*Omega2)
Torque3 = I*a^2*(Rotor3.*Omega3)
%Average Torque, N.m
T1 = sum (Torque1)/4; % summate the torque array and divide by 4
T2 = sum (Torque2)/4;
T3 = sum (Torque3)/4;
Tau1 = [0 T1 T2 T3]

%Theoretical Results
Tau2 = BM*g*arm_length % N.m

%Plots
plot (BM, Tau1, 'ro'),grid on
xlabel('Balance mass, kg', 'FontSize',18)
ylabel('\tau, N.m', 'FontSize',18)
title('Couple for different balance masses', 'FontSize',20)
hold
plot (BM, Tau2, '-go')
legend('Exp. \tau, N.m','Theory \tau, N.m',2)

%Define Best Fit Lines
P1 = polyfit(BM,Tau1,1);
f1 = polyval(P1,BM);
P2 = polyfit(BM,Tau2,1);
f2 = polyval(P2,BM);
plot (BM, f1, '-r', BM, f2, '-g')
axis([0 0.31 0 0.46])

%Plot of precession speed against
%couple for rotor speed of 100 rad/s
Precession = [0 (Omega1(2)+Omega1(3))/2 (Omega2(2)+Omega2(3))/2 Omega3(1)]*a
Tau3 = [0 (Torque1(2)+Torque1(3))/2 (Torque2(2)+Torque2(3))/2 Torque3(1)]
figure, hold off,
plot (Precession, Tau3, 'ro'), grid on, hold on
xlabel('Precession Speed, rad/s', 'FontSize',18)
ylabel('\tau, N.m', 'FontSize',18)
title('Precession Speed and Acting Couple', 'FontSize',20)
text(5.1,0.125,'Rotor Speed, \omega = 100 rad/s','FontSize',20)
```

```
%Define Best Fit Line
P3 = polyfit(Precession, Tau3,1);
f3 = polyval(P3, Precession);
plot (Precession, f3, '-r')
axis([0 8 0 0.5])

%Plot of rotor speed against
%couple for precession speed of 5 rad/s
Rotor = [0 Rotor1(1) Rotor2(2) Rotor3(3)]*a
Tau4 = [0 Torque1(1) Torque2(2) Torque3(3)]
figure, hold off,
plot (Rotor, Tau4, 'ro'), grid on, hold on
xlabel('Rotor Speed, rad/s', 'FontSize',18)
ylabel('\tau, N.m', 'FontSize',18)
title('Rotor Speed and Acting Couple', 'FontSize',20)
text(90,0.125,'Precession Speed, \Omega = 5 rad/s','FontSize',20)

%Define Best Fit Line
P4 = polyfit(Rotor, Tau4,1);
f4 = polyval(P4, Rotor);
plot (Rotor, f4, '-r')
%axis([0 160 0 0.45])
```